



Victoria Math Circle - Session 1

SFFT & Diophantine Exploration

Topic: Simon's Favorite Factoring Trick (SFFT) in Contest Math

Core Identity: $xy + ax + by + ab = (x+b)(y+a)$

Got an elegant solution or solved a Take-Home Challenge? Share your work/draft in our Discord (#solution-share)! Selected solutions will be formatted in LaTeX and published in the official Victoria Math Circle Open-Source Archive on GitHub with full name & school credit. (Sharing is completely optional, but every contribution helps build our open-source community—we'd love to feature your ideas!)

If you didn't come to our first session (you're missing out by the way) and want to share your work, go to our website victoriamathcircle.org to join our discord.

In-session problem set

1. Find the number of pairs of positive integers (x, y) such that:

$$xy + 3x + 2y = 2026$$

(~AMC 10 level)

2. A rectangle with integer side lengths a and b has the property that its area is numerical equal to twice its perimeter. Find the sum of all possible values of $a + b$.

(~AMC 11 level)

3. Two high school students, Alex and Beth, are pooling their savings to watch the movie, Niulai. If Alex gives Beth \$12, Beth will have three times as much money as Alex. If Beth instead gives Alex \$12, Alex will have k times as much money as Beth, where k is a positive integer. Find the maximum possible amount of money Beth could have initially.

(~Cayley level)

4. Find the number of ordered pairs of positive integers (x, y) that satisfy the equation:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$$

such that $x < y$.

(~AIME level)

5. A right triangle has legs of integer lengths a and b , and hypotenuse of length c . If the area of the triangle is equal to its perimeter, find all possible ordered pairs (a, b) with $a \leq b$
- (~AIME level)

6. Determine all pairs of positive integers (m, n) such that $mn - 2m - 3n$ is a prime number p , and $m + n = 16$.
- (~COMC level)

7. Find all triples of positive integers (x, y, z) such that:

$$xy + yz + zx + x + y + z = 2026$$

such that $x \leq y \leq z$.

(~COMC level)

Take-Home challenge set

8. Find all ordered triples of positive integers (a , b , c) with $a < b < c$ that satisfy:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

9. Find all pairs of integers (x , y) that satisfy the equation:

$$x^2y - 3x^2 - 4y + 11 = 0$$

(Hint: Group terms to isolate y or transform it into a polynomials-based SFFT structure.)